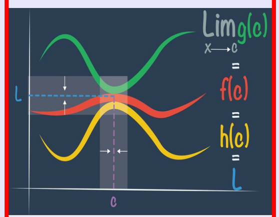
Calculus I

Lecture 1



Feb 19-8:47 AM

Function

every input has only one output

Domain — x (input) Range — y (output)

Function notation $y = f(x)$

ex: $f(x) = \frac{2}{3}x + 1 \longrightarrow f(x) = mx + b$

Slope - Int Form

$f(0) = \frac{2}{3}(0) + 1 = 1$

$m = \frac{2}{3}$ $\frac{\text{Rise}}{\text{Run}}$

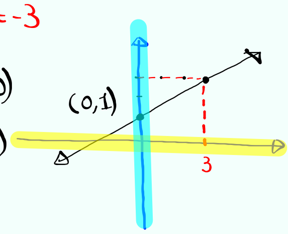
$f(3) = \frac{2}{\cancel{3}}(\cancel{3}) + 1 = 2 + 1 = 3$

y-Int $(0, 1)$

$f(-6) = \frac{2}{\cancel{3}}(\overset{-2}{\cancel{-6}}) + 1 = -4 + 1 = -3$

Domain → input → $(-\infty, \infty)$

Range → output → $(-\infty, \infty)$



Jun 16-8:07 AM

Consider $f(x) = x^2$

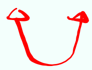
1) $f(0) = 0^2 = \boxed{0}$

Do not use Ø for Zero.

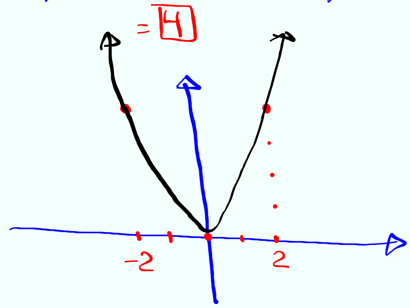
2) $f(2) = 2^2 = \boxed{4}$

3) $f(-2) = (-2)^2 = 4$

4) Graph



Parabola



5) Domain $(-\infty, \infty)$

6) Range $[0, \infty)$

7) Simplify

$$\frac{f(x) - f(2)}{x-2} = \frac{x^2 - 4}{x - 2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \boxed{x+2}$$

$A^2 - B^2 \quad (A-B)(A+B)$

Jun 16-8:14 AM

Consider $f(x) = \sqrt{x}$

1) $f(0) = \sqrt{0} = \boxed{0}$

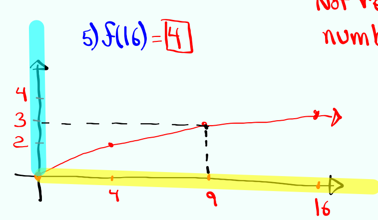
2) $f(4) = \sqrt{4} = \boxed{2}$

3) $f(-4) = \sqrt{-4}$ undefined Not real number

4) $f(9) = \boxed{3}$

5) $f(16) = \boxed{4}$

6) Graph



7) Domain $[0, \infty)$

8) Range $[0, \infty)$

9) Simplify

Conjugate

$$\frac{f(x) - f(9)}{x - 9} = \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

Hint: Rationalize the numerator

$A - B \quad A + B \qquad A^2 - B^2$

$$= \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \frac{(\sqrt{x})^2 - 3^2}{(x-9)(\sqrt{x}+3)}$$

$$= \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} = \boxed{\frac{1}{\sqrt{x}+3}}$$

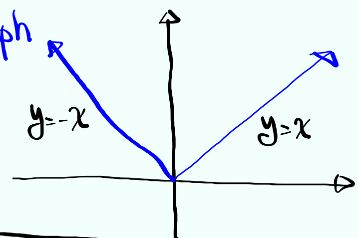
Jun 16-8:27 AM

Consider $f(x) = |x|$

1) $f(0) = |0| = 0$ 2) $f(1) = 1$ 3) $f(-1) = 1$

4) $f(3) = 3$ 5) $f(-3) = 3$

6) Graph



7) Discuss Domain & Range

$(-\infty, \infty)$, $[0, \infty)$

$$\boxed{|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}}$$

Jun 16-8:40 AM

Consider $f(x) = \frac{1}{x}$ Reciprocal Function

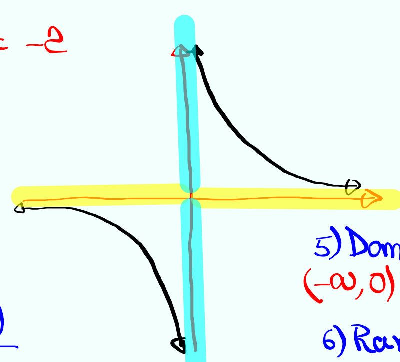
1) $f(0)$ undefined

$x \neq 0$

2) $f(2) = \frac{1}{2}$

3) $f\left(\frac{-1}{2}\right) = \frac{1}{-1/2} = -2$

4) Graph



5) Domain

$(-\infty, 0) \cup (0, \infty)$

6) Range

$(-\infty, 0) \cup (0, \infty)$

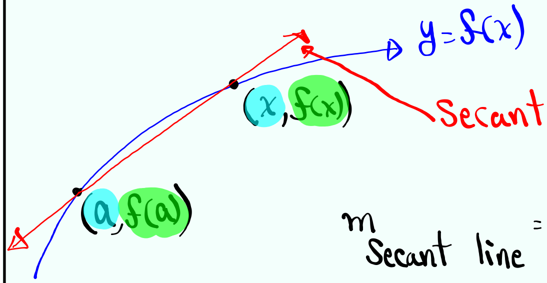
7) Simplify

$$\frac{f(x) - f(1)}{x - 1}$$

$$= \frac{\frac{1}{x} - 1}{x - 1} = \frac{x\left(\frac{1}{x} - 1\right)}{x(x-1)} = \frac{1 - x}{x(x-1)} = \frac{-1\cancel{(x-1)}}{x\cancel{(x-1)}} = \boxed{\frac{-1}{x}}$$

LCD $= x$

Jun 16-8:46 AM

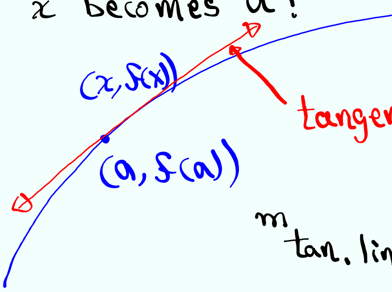


Delta

$$m_{\text{Secant line}} = \frac{\overbrace{\text{changes in } y\text{'s}}^{\Delta y}}{\underbrace{\text{changes in } x\text{'s}}_{\Delta x}}$$

what happens if/when x becomes a?

$$= \frac{f(x) - f(a)}{x - a}$$



$$m_{\text{tan. line}} = \frac{f(x) - f(a)}{x - a}$$

if $x = a$.

Jun 16-8:59 AM

Find $\boxed{\text{slope}}$ of the $\boxed{\text{tangent line}}$ to the graph of $\boxed{f(x) = x^3}$ at $\boxed{x = 2}$.

$$m_{\text{tan. line}} = \frac{f(x) - f(a)}{x - a} = \frac{x^3 - 2^3}{x - 2}$$

$$= \frac{x^3 - 8}{x - 2}$$

$$= \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{x-2}}$$

$$= x^2 + 2x + 4$$

at $x = 2$

$$m = 2^2 + 2(2) + 4 = \boxed{12}$$

Hint:

$A^2 + B^2$ prime

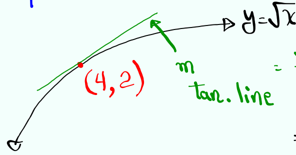
$A^2 - B^2 = (A + B)(A - B)$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

Jun 16-9:07 AM

find slope of the tangent line to the graph of $f(x)=\sqrt{x}$ at $x=4$.



$$m_{\text{tan. line}} = \frac{f(x)-f(a)}{x-a} \quad \text{when } x=a$$

$$= \frac{\sqrt{x}-\sqrt{4}}{x-4}$$

$$= \frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$$

Rationalize the numerator

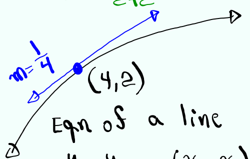
$$= \frac{(\sqrt{x})^2-(2)^2}{(x-4)(\sqrt{x}+2)}$$

$$= \frac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{x}+2)}$$

$$= \frac{1}{\sqrt{x}+2}$$

for $x=4$

$$m = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$



Eqn of a line

$$y-y_1 = m(x-x_1)$$

$$y-2 = \tfrac{1}{4}(x-4)$$

$$y = \tfrac{1}{4}x - \tfrac{1}{4}\cdot 4 + 2$$

$$y = \tfrac{1}{4}x - 1 + 2 \Longrightarrow \boxed{y = \tfrac{1}{4}x + 1}$$

Jun 16-9:15 AM

Consider $f(x) = \dfrac{2}{x-2}$

1) $f(0) = \dfrac{2}{0-2} = \dfrac{2}{-2} = \boxed{-1}$

Y-Int $(0,-1)$

2) Solve $f(x)=0$ $\quad y=0$

$\dfrac{2}{x-2} = 0 \rightarrow$ No Solution, No x-Int

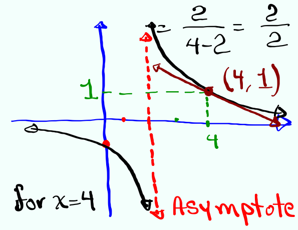
3) Discuss domain

$x-2 \neq 0 \quad x \neq 2$

$(-\infty,2)\cup(2,\infty)$

4) find $f(4) = \dfrac{2}{4-2} = \dfrac{2}{2} = 1$

5) Graph



$$m_{\text{tan. line}} = \frac{f(x)-f(4)}{x-4} \quad \text{for } x=4$$

$$= \frac{\frac{2}{x-2}-1}{x-4} = \frac{(\cancel{x-2})\cdot\frac{2}{\cancel{x-2}} - (x-2)\cdot 1}{(x-2)\cdot(x-4)}$$

LCD $= x-2$

$$= \frac{2-(x-2)}{(x-2)(x-4)}$$

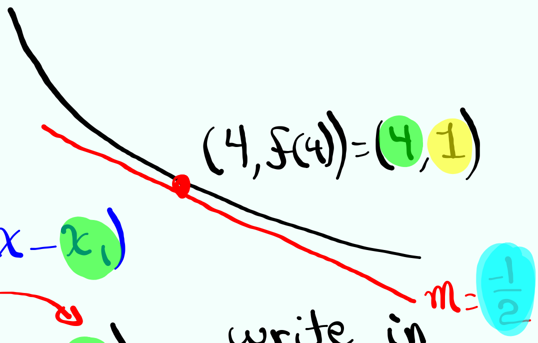
$$= \frac{2-x+2}{(x-2)(x-4)} = \frac{\cancel{4-x}}{(x-2)(\cancel{x-4})}\quad -1$$

$$m_{\text{tan. line}} = \frac{-1}{x-2} \quad \text{for } x=4$$

$$m = \frac{-1}{4-2} = \boxed{\frac{-1}{2}}$$

Jun 16-9:50 AM

what about eqn of tan. line at $x=4$?



$y - y_1 = m(x - x_1)$

$y - 1 = \frac{-1}{2}(x - 4)$

$y - 1 = \frac{-1}{2}x - \frac{1}{2}(-4)$

$y = \frac{-1}{2}x + 2 + 1$

write in slope-Int. Form

$y = mx + b$

$$\boxed{y = \frac{-1}{2}x + 3}$$

Jun 16-10:01 AM

Introduction to limits

Notation

$\lim_{x \to a^+} f(x) = L_1$

from right

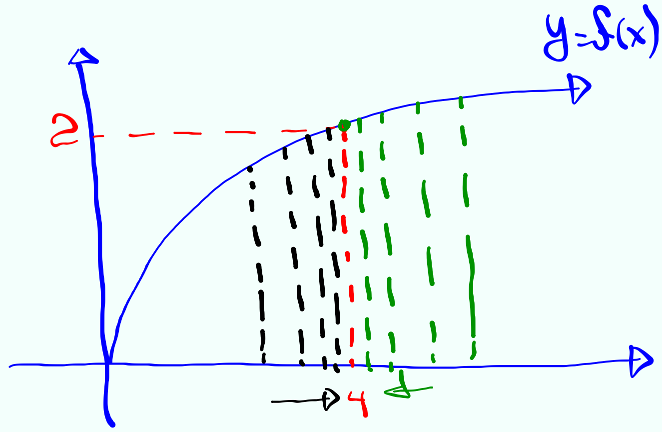
$\lim_{x \to a^-} f(x) = L_2$

from left

if $L_1 = L_2 \Rightarrow \lim_{x \to a} f(x) = L$

Jun 16-10:09 AM

$f(x) = \sqrt{x}$

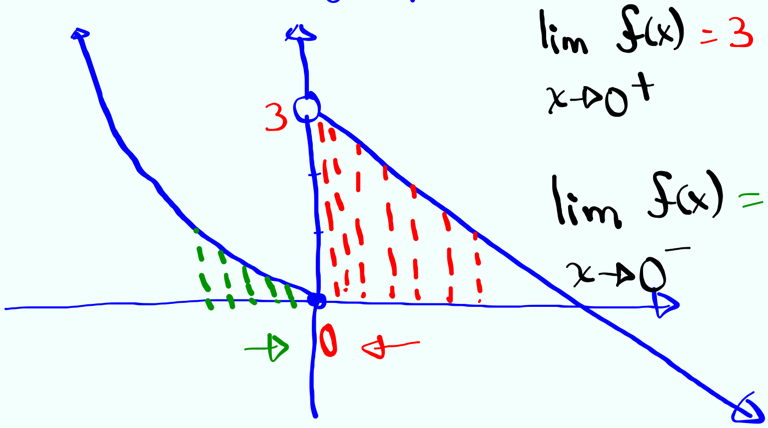


$$\lim_{x \to 4^+} \sqrt{x} = 2 \qquad \lim_{x \to 4^-} \sqrt{x} = 2$$

$$\boxed{\lim_{x \to 4} \sqrt{x} = 2}$$

Jun 16-10:11 AM

Consider the graph of f(x) below



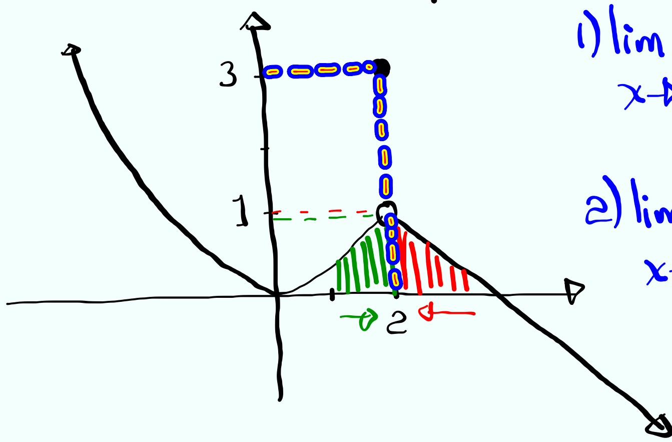
$$\lim_{x \to 0^+} f(x) = 3$$

$$\lim_{x \to 0^-} f(x) = 0$$

Since $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$, then $\lim_{x \to 0} f(x)$ DNE

Jun 16-10:14 AM

Consider the graph of $f(x)$ below



1) $\lim\limits_{x \to 2^+} f(x) = 1$

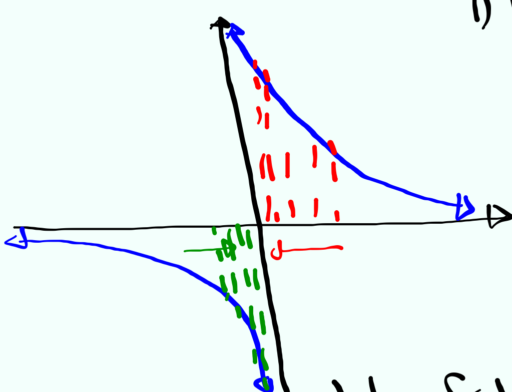
2) $\lim\limits_{x \to 2^-} f(x) = 1$

3) $\lim\limits_{x \to 2} f(x) = 1$

4) find $f(2) = 3$

Jun 16-10:18 AM

Graph of $f(x) = \frac{1}{x}$ is given below:



1) $\lim\limits_{x \to 0^+} f(x) = \boxed{\infty}$

2) $\lim\limits_{x \to 0^-} f(x) = \boxed{-\infty}$

3) $\lim\limits_{x \to 0} f(x)$

DNE

4) $f(0)$

undefined

5) $\lim\limits_{x \to \infty} f(x) = 0$

6) $\lim\limits_{x \to -\infty} f(x) = 0$

Jun 16-10:23 AM

Strategies to evaluate limits

1) Plug it in, and hope for the best.

$$\lim_{x \to 2} (x^2 + 3x) = (2)^2 + 3(2) = \boxed{10}$$

$$\lim_{x \to 2} \frac{x+3}{x-1} = \frac{2+3}{2-1} = \frac{5}{1} = \boxed{5}$$

2) If we get an indeterminate form $\frac{0}{0}$, then simplify by factoring, rationalizing, or using LCD.

$$\lim_{x \to 2} \frac{x^2 - 2x}{x-2} = \frac{2^2 - 2(2)}{2-2} = \frac{4-4}{2-2} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{x^2 - 2x}{x-2} = \lim_{x \to 2} \frac{x\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x \to 2} x = \boxed{2}$$

Jun 16-10:28 AM

Evaluate $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \frac{1-1}{\sqrt{1}-1} = \frac{1-1}{1-1} = \frac{0}{0}$ I.F.

$$\lim_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x})^2 - 1^2}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{x-1}} = \lim_{x \to 1} (\sqrt{x}+1) = \sqrt{1}+1$$

$$= \boxed{2}$$

Jun 16-10:36 AM

Evaluate

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} = \frac{\frac{1}{2} - \frac{1}{2}}{2-2} = \frac{0}{0} \text{ I.F.}$$

$2x \cdot \frac{1}{x} = 2$

$2x \cdot \frac{1}{2} = x$

Use LCD to simplify

$\hookrightarrow 2x$

$$\lim_{x \to 2} \frac{2x\left(\frac{1}{x} - \frac{1}{2}\right)}{2x(x-2)} = \lim_{x \to 2} \frac{\overset{-1}{\cancel{2 - x}}}{2x\cancel{(x-2)}}$$

$$= \lim_{x \to 2} \frac{-1}{2x} = \frac{-1}{2(2)} = \boxed{\frac{-1}{4}}$$

Jun 16-10:41 AM

Class QZ 1

Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

to solve $2x^2 + 3x - 5 = 0.$

$\boxed{\text{Box Your Answer}}$

$a = 2, \ b = 3, \ c = -5$

$b^2 - 4ac = 3^2 - 4(2)(-5) = 49$

$$x = \frac{-3 \pm \sqrt{49}}{2(2)} = \frac{-3 \pm 7}{4}$$

$x = \frac{-3-7}{4} = \frac{-10}{4} = \boxed{\frac{-5}{2}}$

$x = \frac{-3+7}{4} = \frac{4}{4} = \boxed{1}$

Jun 16-10:46 AM